The ecological dimension in the teaching of modelling at university level

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THE ECOLOGICAL DIMENSION IN THE TEACHING OF MATHEMATICAL MODELLING AT UNIVERSITY

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Abstract – Recent research on the teaching and learning of mathematical modelling highlights the existence of strong institutional constraints impinging on the large-scale dissemination of mathematics as a modelling activity−in current educational systems at all school levels. The study of these constraints and the way new teaching proposals can overcome these, what we call the *ecology* of modelling, appears as a necessary step for mathematics education research. Within the framework of the anthropological theory of the didactic, the hierarchy of levels of didactic codetermination is used to identify and analyse this ecology, not only to deal with the variety of constraints appearing in classroom activities, but mainly to know at what level action is necessary. A teaching proposal in terms of *study and research paths* in tertiary education shows new possibilities to overcome some of the constraints that hinder the development of mathematics as a modelling activity.

Keywords: mathematical modelling, ecology, applicationism, institutional constraints, study and research paths

LA DIMENSION ECOLÓGICA EN LA ENSEÑANZA DE LA MODELIZACIÓN MATEMÁTICA EN LA UNIVERSIDAD

Resumen – Investigaciones recientes sobre la enseñanza y el aprendizaje de la modelización matemática han puesto en evidencia la existencia de fuertes restricciones sobre la difusión generalizada y a largo plazo de la matemática como herramienta de modelización en los sistemas de enseñanza actuales. El estudio de estas restricciones y de las condiciones que permiten superarlas, lo que llamamos la *ecología* de la modelización matemática, aparece así como una etapa inevitable de la investigación didáctica. En el marco de la teoría antropológica de lo didáctico, la utilización de una jerarquía de niveles de codeterminación didáctica permite identificar y analizar la emergencia de estas restricciones, no solo para abordarlas en su diversidad, sino sobre todo para saber en qué nivel es necesario actuar. Diseñados en base a este análisis ecológico, los *recorridos de estudio e investigación* aparecen entonces como un dispositivo didáctico apropiado para superar las restricciones que limitan la vida de la matemática como herramienta de modelización en los actuales

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Palabras-claves: modelización matemática, ecología, aplicacionismo, restricciones institucionales, recorridos de estudio e investigación

LA DIMENSION ÉCOLOGIQUE DANS L'ENSEIGNEMENT DE LA MODÉLISATION MATHÉMATIQUE À L'UNIVERSITÉ

Résumé – Des recherches récentes sur l'enseignement et l'apprentissage de la modélisation mathématique mettent en évidence l'existence de fortes contraintes institutionnelles sur la diffusion généralisée et à long terme des mathématiques comme outil de modélisation dans les systèmes d'enseignement actuels. L'étude de ces contraintes et des conditions qui permettent de les dépasser, ce que nous appelons l'*écologie* de la modélisation mathématique, apparaissent ainsi comme une étape inévitable de la recherche en didactique. Dans le cadre de la théorie anthropologique du didactique, l'utilisation d'une hiérarchie de niveaux de codétermination didactique permet d'identifier et d'analyser l'émergence de ces contraintes, non seulement pour les atteindre dans leur variété, mais surtout pour savoir à quel niveau il est nécessaire d'agir. Dans ce contexte, les *parcours d'étude et de recherche* apparaissent comme un dispositif didactique approprié pour dépasser certaines des contraintes qui entravent la vie de la mathématique comme outil de modélisation dans les systèmes d'enseignement universitaires actuels.

Mots-Clés : modélisation mathématique, écologie, applicationnisme, contraintes institutionnelles parcours d'étude et de recherche.

INTRODUCTION

In spite of all the progress made in the educational field of 'Modelling and Applications' to implement controlled processes of teaching and learning mathematics as a modelling tool, recent research highlights the existence of strong institutional constraints on the large-scale dissemination of modelling practices in current educational systems at all school levels (section 1). This paper addresses this problem, reformulating the question within the framework of the Anthropological Theory of the Didactic (ATD) in terms of the 'ecology' of mathematical modelling activities. A hierarchy of levels of didactic codetermination is proposed as a tool to identify and analyse the emergence of these constraints, not only to deal with the variety of constraints in classroom activities, but also to know at what level it is necessary to act to improve the conditions for mathematical modelling to exist as a regular activity (section 2).

We tackle the problem of the ecology of mathematical modelling in the special settings of mathematics teaching in natural science faculties and by means of a double methodology. On the one hand, we use a naturalistic observation of the educational institutions and situations on which we focus, especially when considering the constraints coming from the dominant epistemology and pedagogy at university level (section 3). On the other hand, and in synchrony with the former, we carry out an analysis with participative intervention consisting in the local implementation of a teaching activity named *study and research path* (or *study and research course*)[4] along five academic years (section 4). Because the university system and the way it organises mathematics teaching is familiar, most of its features become natural to the observer and, thus, transparent or invisible. The modification of this system shows how it resists to the changes introduced, bringing to light the specific constraints that limit this development and that could, in the long run, put its viability at risk.

INTEGRATING MATHEMATICAL MODELLING INTO CURRENT EDUCATIONAL SYSTEMS

Over the last few decades, several investigations from different theoretical perspectives have shown that mathematical modelling activities can exist at school under suitable conditions, at all levels and in almost all curricular contents. It is worth mentioning the 14th ICMI

[4] The French expression 'parcours d'étude et de recherche' ('recorridos de studio e investigación', in Spanish) was initially translated into 'research and study course'. In a recent work, Yves Chevallard (2012) proposed the term 'study and research path'. We are adopting this last translation to avoid the polysemy of the word 'course' so largely used in the educational context as a set of lessons in a curriculum.

Study, *Applications and modelling in mathematics education* (Blum, Galbraith, Henn & Niss 2002 and 2007); the two 2006 special issues of *ZDM - The International Journal on Mathematics Education* (volumes 38(2) and 38(3)) and the International Conferences on the Teaching of Mathematical Modelling and Applications (ICTMA) which have been held biannually since 1983.

Besides all the progress made in establishing modelling as a well-established activity in mathematics teaching and learning processes, the problem of the large-scale dissemination of these processes has recently been addressed as both an urgent and intricate task. Some researchers have pointed out the existence of difficulties or strong limitations hindering the inclusion and permanent subsistence of mathematical modelling practices in the classroom. Related to this state of things, Burkhardt (2008) emphasizes the existence of two realities: on the one hand, the good progress and encouraging results in research with respect to teaching modelling and applications; on the other hand, the difficulties of its *large-scale dissemination in the classroom*. He quite harshly states:

> [W]e know how to teach modelling, have shown how to develop the support necessary to enable typical teachers to handle it, and it is happening in many classrooms around the world. The bad news? 'Many' is compared with one; the proportion of classrooms where modelling happens is close to zero. (p. 2091)

Moreover, Galbraith (2007) describes the situation using a suggestive metaphor:

> When the low hanging fruit is also sweet, the incentive to search the higher branches is diminished further. In the case of applications and modelling a shared excitement unites many who have enthused about early experiences in the field, for example when students unleash latent power that for whatever reason had remained fettered in their previous mathematical life. However, this very exhilaration can work against further progress, both individually, and particularly at a system level, by creating a sense of adequate achievement that obscures the reality that there is so much more to do. (p. 78)

To depict the difficulties encountered in the dissemination of mathematical modelling practices, many researchers use expressions such as, among others, 'counter-arguments' (Blum 1991), 'teachers' beliefs as obstacles' (Kaiser 2006, p. 399 and Kaiser & Maaß 2007, pp. 99-105), 'dilemmas' (Blomhøj & Kjeldsen 2006, pp. 175-176) or 'barriers' (Burkhardt 2006, pp. 190-193), pointing out a new direction of research, which moves from a first phase focused on the problem of the design, implementation and analysis of modelling practices to the *study of the constraints* that affect the existence, permanence and evolution of these practices. But most of these investigations are focused on teachers and their daily practice in classrooms:

> Specifically the result that teachers and their beliefs about mathematics must be regarded as essential reasons for the low realisation of applications and modelling in mathematics teaching (Kaiser & Maaß 2007, p.10)

Some authors discuss the existence of more generic limitations that hinder mathematical modelling practices, for instance, Burkhardt (2006, pp. 190-193) outlines and discusses the existence of 'barriers' that obstruct a large-scale implementation of modelling, such as the systemic inertia, the unwelcome complication of the real world in many mathematics classrooms, and the limited professional developments of teachers. To overcome these barriers and many others still unknown, he refers to 'levers' (such as changes in curriculum descriptions supported by well-engineered materials to support assessment, teaching and professional development, etc.) that may provide some promising progress in this field.

As for Doerr (2007), she refers to more general constraints related to traditional pedagogies:

> Teaching mathematics through modelling provides substantial challenges to our current ideas about pedagogy […]. A modelling approach to teaching mathematical modelling calls for a major reversal in the usual roles of teachers and students. Students need to do more evaluating of their own ideas and teachers need to create opportunities where this evaluation can productively occur. Current research in the preparation and development of teachers in taking on these roles is limited. (p.78)

Michelsen (2006) points out an even more general barrier when he questions the common separate vision of scientific disciplines, and states that traditional delimitations between disciplines suppose a constraint for the development of applied activities:

> The challenge is to replace the current monodisciplinary approach, where knowledge is presented as a series of static facts disassociated from time with an interdisciplinary approach, where mathematics, science, biology, chemistry and physics are woven continuous together. (p. 269)

To conclude, we can say that, initially, research on the teaching and learning of mathematical modelling focused on the design and analysis of modelling activities (in terms of 'competences' or by means of what is called the 'modelling cycle' and its multiple variations, see, for instance, Blum & Leiß 2007), paying special attention to its use in the design of teaching and learning activities and to the analysis of students' difficulties. It has currently moved to a second phase in which many researchers start to approach *the problem of the institutional constraints* that hinder the possible large-scale dissemination of mathematical modelling activities at all school levels. Our work aims to contribute to this recently growing stream of research.

THE PROBLEM OF MATHEMATICAL MODELLING FROM THE POINT OF VIEW OF THE ANTHROPOLOGICAL THEORY OF THE DIDACTIC

In what follows, we will use the framework of the Anthropological Theory of the Didactic (ATD) and the vision of mathematical modelling proposed by García, Gascón, Ruiz-Higueras & Bosch (2006) according to the ATD *general epistemological model of mathematical activity*.

The ATD uses the notion of *mathematical praxeology* or *organization* as a fundamental tool to describe and analyse mathematical activity. In a mathematical organisation, two inseparable aspects can be distinguished: the block of the *practice* (or *praxis*) that consists of *types of problematic tasks* and *techniques* to tackle these tasks. Linked to this block, a reasoned discourse (*logos*) arises about the practice, whose function is to provide a description, explanation and justification of the practice. This discourse is split into two components: the first level of description is called the *technology* (the *logos* of the *techne*); the second level, the *theory*, contains the foundations of the technology, thus providing a second-order base of the practice.

Several research works within this framework have analysed and described mathematical modelling activities from this approach (Chevallard 1989, 1992; Chevallard, Bosch & Gascón 1997; García et al. 2006). All these authors assume that doing mathematics essentially consists in the activity of producing, transforming, interpreting and arranging mathematical models and it thus establishes a continuous link between mathematical modelling and the construction of mathematical concepts.

In accordance with García et al. (op. cit.), who use this framework to generalise the notion of the 'modelling cycle' proposed by Blum & Leiß (2007), we will consider modelling as *a process of reconstruction of mathematical praxeologies* that become progressively *broader* and more *complex* (from point, to local and regional praxeologies) (Bosch, Fonseca & Gascón 2004). This process starts with the study of a question arising in an extra-mathematical situation, the initial system, and leads to the construction (or consideration) of mathematical praxeologies that can act as a *model* of the considered initial system in the sense that properties of the 'system' can be derived from those of the model. This work usually creates new needs and new questions, which require the construction (or consideration) of new models, the previous model thus acting as a 'system' of this new modelling process (Serrano, Bosch & Gascón 2010). Mathematical modelling can therefore be considered as a process of *connecting* mathematical concepts or, more properly, praxeologies. Furthermore, from this viewpoint, intra-mathematical modelling—that is, the process of modelling mathematical systems—appears as a particular case of mathematical modelling.

The ATD vision of mathematical modelling is thus characterised by the praxeological structure of models and systems, the inclusion of intra-mathematical modelling and the power of connecting mathematical praxeologies. In this sense, mathematical modelling cannot only be considered as an aspect or modality of mathematical activity but has to be placed at the core of it. Considering mathematical modelling as an intrinsic part of any mathematics study process appears as an essential aspect of the research problem that we are presenting in the following section.

The ecological dimension of mathematical modelling activities

The brief account of the development of research field of *Modelling and applications* presented in the introduction shows how the focus was initially on describing the modelling activity, the corresponding design of activities and the analysis of students' difficulties. Later on, this research problem was extended towards the study of the constraints limiting the possible generalisation of modelling practices in current educational systems, with the aim of identifying some conditions that could encourage its effective and large-scale dissemination.

In the ATD, we refer to these conditions and constraints as the *ecological dimension of mathematical modelling* (Gascón 2011; Barquero, Bosch & Gascón 2013). Since the works of Rajoson (1988) and Artaud (1998), the study of the *ecology* of mathematical objects refers to the analysis of the network of interrelations and functions of each mathematical object (notion, technique, process, etc.) in a given mathematical activity. The ATD uses the specific term of *institutional life* to refer to the ways of doing and thinking that are normally developed in a given institutional environment. For instance, the teaching of classical languages such as Latin or Greek, which had a 'radiant life' in the old days, is now in most countries almost an 'endangered species'. In the same way, the ecological conditions for the teaching of English as a foreign language can be considered as highly positive and 'healthy' in many European countries. The origin of the *ecological approach*, which was first applied to mathematical objects and practices before being extended to a wider institutional perspective, can be found in the study of the process of *didactic transposition* and its related phenomena (Chevallard 1985, see also Bosch & Gascón 2006). It consists in formulating didactic problems in terms of the *conditions* that enable, favour or facilitate the development of certain praxeologies in a given institutional environment (primary school, university, the society at large, etc.) and the *constraints* hindering, or even impeding, their realization and development. The notions of *condition* and *constraint* are relative to the position assumed by a person in a given institution. A *condition* becomes a *constraint* when it cannot be modified by the person in this position, at least not in the short run. For instance, the way of grouping students by age or the distribution of contents into subjects are constraints related to the positions of teachers and students, even if they can be modified from other positions, from educational authorities or politicians, for example.

This ecological approach has led to several works studying the evolution of the conditions related to the introduction of different mathematical objects in the curricula of different school levels, such as the work by Artaud (1998) about the role of mathematics in economics, Assude (1996) about square roots at secondary level, Coulange (2001) about systems of equations at secondary level, Chambris (2010) about operations with numbers and quantities at primary level, to mention but a few.

In our case, dealing with the problem of how to 'normally' integrate modelling into current education systems, we postulate that it is essential to carry out an in-depth analysis of all the necessary conditions required and the constraints that hinder its large-scale development in educational institutions. We formulate this research problem in the following terms:

What kinds of limitations and constraints exist in our current educational systems that prevent mathematical modelling from being widely incorporated in daily classroom activities?

Taking into account this specific ecology of mathematical modelling, what kind of conditions—in terms of didactic activities—could help a large-scale integration of mathematical modelling at school?

Within the framework of the ATD, most of the research related to mathematical modelling and teaching practices takes into account the problem of the *ecology* of didactic organisations (Chevallard 1989, Artaud 2007, Bolea, Bosch & Gascón 2004, Barquero 2009, Barbé, Bosch, Espinoza & Gascón 2005). Before developing this point, we will briefly present the levels of didactic codetermination, a key notion introduced by Chevallard (2002) to analyse the different kinds of conditions and constraints that affect teaching and learning processes (see also Artigue & Winsløw 2010).

The hierarchy of levels of didactic codetermination

To study the ecology of mathematical practices that exist (or could exist) in a teaching institution and the possible ways of constructing them (the didactic organisations), Chevallard introduced a hierarchy of levels of didactic codetermination that take into account the mutual incidence between mathematical contents and the way of organising their teaching. It consists in the following sequence (Chevallard 2002, p. 10):

Civilization ⇔ Society ⇔ School ⇔ Pedagogy
⇔ Discipline ⇔ Domain ⇔ Sector ⇔ Theme ⇔ Question

Figure 1. – Scale of levels of didactic codetermination

This hierarchy goes from the most generic level, the *Civilization*, to the most concrete one, the *subjects* or the specific mathematical *questions* that are at the core of the didactic process considered. The levels going from *Discipline* to *Questions* are referred to as *specific* levels. The conditions and constraints affecting a given process of teaching and learning may vary from one discipline to another, and are also relative to the particular way the discipline is organised or structured in subdivisions (called *Domains*, *Sectors* and *Themes*) in the educational institution considered. For instance, in Spain, a first-year university course of mathematics in a natural science degree is usually structured into three 'domains'—calculus, linear algebra and differential equations—that are in turn divided into 'sectors' (such as: single-variable calculus, multi-variable calculus, etc.) containing different 'themes' to which every subject or question to study belongs (limits, differentiation, determinants, etc.) At secondary school level, the domains are different and can change over time, for instance because of a curricular reform: the classical division into 'arithmetic, algebra, geometry' that used to organise mathematical contents for ages no longer prevails and has been replaced by other divisions such as 'numbers and measure, functions, geometry, statistics' or 'change and relations, space and form, statistics, measure, number', etc. Putting into question the choice of decisions taken to organise mathematical teaching (and learning) at these specific levels is useful to analyse the constraints derived from the didactic transposition process (Chevallard 1985) and breaks up with the 'transparency' of taught mathematical contents.

The *upper levels* of codetermination refer to the more general constraints derived from the way our societies, through school institutions, organise the study of disciplines. They concern the status and functions traditionally assigned to educational contents and the general way teaching and learning activities are organised. It is obvious that an educational institution establishes common conditions that particularly affect what the teacher and students can do in the classroom, for instance: the amount of hours and sessions assigned to each discipline or subject matter, the possibilities for disciplines to interact more or less easily, the way students are grouped (by age, by level, by gender, etc.), the organisation of the school space, etc. All those *conditions* and *constraints* belong to the *school* level, while the *pedagogical* level refers to those only affecting the teaching and learning of disciplines. The way disciplines are grouped, valued, linked and diffused belongs to this level: for instance, the choice of an interdisciplinary way of studying questions or the way of presenting disciplines as independent. The delimitation of a discipline or subject itself belongs to this level. Very close to the previous levels, the *Society* and *Civilization* levels concern the way our societies and civilizations understand the rationale, functions, aims, etc. of school instruction.

Conditions and methodology of the experimental analysis

In our study of the institutional ecology of mathematical modelling, our research focuses on the case of a first-year course of mathematics in a natural sciences programme. A double methodology was developed. On the one hand, we used a *naturalistic* observation *(without intervention)* of the educational institutions on which we focus to approach the constraints that appear at the generic levels of didactic codetermination (see figure 1). These constraints mainly derive from the *dominant epistemology* and *pedagogy* in scientific university communities.

On the other hand, we made an *observation with participative intervention* to confirm the presence of these constraints and also to discover new constraints appearing at the specific levels of didactic codetermination. These levels are more closely related to the fact that the taught subject matter is mathematics and mathematical modelling. This intervention consisted in the local implementation of the new teaching activity called *study and research paths* (SRP) (Chevallard 2006 and 2012). SRP were introduced to facilitate the inclusion of mathematical modelling in educational systems and, more importantly, to explicitly situate mathematical modelling problems at the centre of teaching and learning practices (see Barquero, Bosch & Gascón 2008 and 2011a). The particular SRP considered in this paper focuses on the study of population dynamics and was developed throughout five academic years (from 2005/06 to 2009/10).

Following Barquero et al. (*to appear*), we will explain in what sense the learning conditions introduced by SRP respond to some of the constraints imposed by the dominant epistemology and pedagogy on school scientific activities in general and on school mathematical modelling in particular. These new conditions can be understood as working principles (or assumptions) that can be taken into account for the possible design, implementation and analysis of a SRP. Meanwhile, the modification of the traditional conditions produced by SRP shows how the system 'resists' the changes introduced, bringing to light the specific constraints that limit this development and should be taken into account to ensure the long-term viability of this way of teaching mathematics as a modelling tool.

CONSTRAINTS AT THE GENERIC LEVELS OF DIDACTIC CODETERMINATION

Taking into account the generic levels of the scale, from Civilization to Discipline, we will focus on describing some constraints related, in the first place, to what may be called the *dominant epistemology*, that is, the way our society, the university as an institution and, more particularly, the community of university teachers and students, understand what mathematics is and what its relation is to natural sciences. Secondly, we will consider some constraints derived from what we call the *dominant pedagogy*, which is how these institutions, and universities, understand

and organise the teaching and learning processes of the disciplines (mathematics in particular).

Our first hypothesis is that one of the main components of this dominant epistemology appears in a concrete way of interpreting, describing and conceptualizing the relation between mathematics and natural sciences, that we call *applicationism*. It roughly consists in considering that mathematics has to be introduced by itself, having its own rationale, before being applied to extra-mathematical situations. For example, a manifestation of applicationism appears in many university mathematics courses for natural sciences, where the study of topics such as ‘population dynamics’, ‘infectious diseases’, ‘genetics’, ‘Markov chains’, etc. are located in the Sector (‘differential equations’, ‘matrices’, ‘probability’) under the label of ‘applications’, as if some dynamics laws could exist without any mathematical tool to describe them and, in the same way, as if mathematical tools existed independently of extra-mathematical problems.

Before presenting the main characteristics of such applicationism, it needs to be highlighted that, as a component of the dominant epistemology, it cannot be located in only one of the generic levels of the scale. As a way of interpreting mathematics in general, it belongs to the level of Society (and even Civilization). However, it is difficult to have a general idea of a discipline without taking into account the way this discipline is constructed and diffused. We can hardly separate this concept from School (especially when talking about University) and its lower levels. Nevertheless, we consider that the constraints derived from applicationism come from the generic levels of the scale even if they also appear at the more specific levels.

Constraints related to applicationism as the dominant epistemology at university level

One of the main characteristics of applicationism is that it greatly restricts the notion of *mathematical modelling*. Under its influence, modelling activity is understood and identified as a mere application of previously constructed mathematical knowledge or, in the extreme, as a simple exemplification of mathematical tools in some extra-mathematical contexts artificially built in advance to fit these tools. In Barquero, Bosch & Gascón (2011b and *to appear*), we describe how to characterise and empirically contrast what is and up to what degree applicationism prevails in university institutions using the following indicators:

I_1: *Mathematics is independent of other disciplines* (‘epistemological purification’): Mathematical tools are seen as independent of extra-mathematical systems and they are applied in the same way, independently of the nature of the system considered.

I_2: *Basic mathematical tools are common to all scientists*: all students can follow the same introductory course in mathematics, without considering any kind of specificity depending on their speciality (biology, geology, chemistry, etc.).

I$_3$: *The organisation of mathematics contents follows the logic of the models*: Instead of being developed from considering modelling problems that arise in the different disciplines. It all happens as if there were a unique way of organising mathematical contents and different ways of applying them.

I$_4$: *Applications always come after the basic mathematical training*: The first thing is to learn how to manipulate the mathematical concepts and later learn about their use. The models are built from concepts, properties and theorems of each theme independently of any extra-mathematical system.

I$_5$: *Extra-mathematical systems can be conceived or built without almost any reference to mathematical models.* The belief that natural science can be taught without any mathematics is a consequence.

These indicators were used as a guide for an empirical study to investigate the prevalence of 'applicationism' in the university teaching of natural sciences. This study was based on the analysis of teaching materials (28 syllabi of mathematics in natural sciences[5] university courses of 10 different Spanish universities; 5 international textbooks of mathematics for natural sciences), which constituted a good indicator of the mathematics taught to students. The next step was to look into the discourses of university teachers and researchers when they were asked about the criteria used to design and manage mathematics teaching in natural sciences degrees. For this purpose, we first conducted a survey on the indicators that characterize applicationism. The survey was distributed and answered by 30 teachers and researchers in the departments of Biology, Geology and Environmental Sciences at the Autonomous University of Barcelona (UAB). Moreover, we completed the findings with interviews of four teachers that had participated in the survey, with the objective of explaining and expanding their answers. The results obtained in the survey were, for most questions, quite clear. We next summarize[6] some of the main findings in terms of some important constraints that prevent mathematical modelling from being widely incorporated in university classrooms.

One of the main characteristics of 'applicationism', and that implies one of the strongest constraints on the normal life of mathematical modelling, is that it establishes a *clear distinction between mathematics* and the *rest of natural sciences*. It is furthermore supposed that both 'worlds' evolve with independent logic and without too many interactions. This fact, which partly results from the first three indicators of applicationism [I_1, I_2, I_3], greatly reduces the possible role of mathematics as a modelling tool of scientific systems and even denies the role of mathematics as a key tool to study the problems that arise in

[5] We focused on the particular case of a first year course of mathematics in the degrees of Biology, Geology, Chemistry and Environmental Sciences.

[6] An extended version of this study can be found in Barquero, Bosch & Gascón (2010 and 2013).

extra-mathematical systems. In general, the mathematics taught present a highly stereotyped and crystallized structure that does not mingle with the systems that are modelled and, moreover, the mathematics taught are never 'modified' as a consequence of being applied.

We find several examples of this in the case of the written description of different mathematics courses of first year university scientific programmes in Spain stating that the teaching of mathematics follows a double objective: on the one hand, they strive to give students basic mathematical training; on the other hand, they try to introduce students to mathematical modelling [I_1, I_2]. For instance, in the case of a geology degree (2006/07) at the Barcelona Autonomous University, the following is proposed (our translation):

> This programme has a double objective. The first and most important one is to give students the basic mathematical training focused on linear algebra and on single-variable differential calculus, which will enable them to understand the language of Science. The second aim is to introduce them to the field of Geology, that is, to mathematical modelling, using simple examples that could be analysed by means of previously introduced mathematical tools.

In a more detailed analysis of the mathematical contents of these courses, we noticed they are organised into 'topics', 'themes' or 'modules' based on a main concept (limits, derivatives, integration, linear applications, diagonalisation, ordinary differential equations, etc.) each including a number of definitions, properties, theorems, proofs, several techniques and types of problems. The corresponding study programme is generally structured into three main areas: linear algebra, single-variable differential and integral calculus, and ordinary differential equations [I_3].

Moreover, when teachers were asked about whether mathematics is introduced independently from scientific (physic, chemical, biological, etc.) systems [I_1]: 93% agreed (43% strongly agreed) this was true. A teacher agreed with the fact that mathematics is introduced independently:

> I do not think that mathematics should be introduced linked to geological systems. How could it be done by using only such basic mathematics? I do not find it convenient or pertinent because we do not have to explain why mathematics is needed in each of our courses. Mathematics must be there, for its own sake.

More than 50% of the teachers did not agree with the fact that the mathematics taught in their respective scientific disciplines were any different from the ones taught in the rest of natural sciences. We may thus confirm that the mathematics taught in one particular science is not specific to it [I_2]. Another teacher added:

> I think that the differences between degrees need not be very significant. Mathematical language is unique, and the attraction to mathematics can

> come from different scientific perspectives but it is always the same. Nevertheless, I think that details matter, the type of examples, the adaptations, being able to visualize that those equations can be translated into specific phenomena […].

Teachers agreed that mathematics, which is taught at first year science degrees, presents a highly stereotyped and crystallized structure that does not mingle with the systems that are modelled and, moreover, the mathematics taught are never 'modified' as a consequence of being applied in the construction of such models. In all cases the way of organising its teaching reproduces the internal logic of mathematics itself [I_3]. On this point, another teacher explained:

> I think that the taught contents are still centred on mathematical problems, but I do not know whether this is bad, I find it natural. I said earlier that syllabi did not have to be completely different and that the differences should appear in the examples. What I meant was that I am glad that the mathematics programmes for physicists and for bio-scientists do not contain large differences and share a common body, which cannot be physics, chemistry or biology: it has to be mathematics […]

This radical separation between mathematics and natural sciences makes it very difficult to consider *mathematics as a constitutive tool of natural sciences*. We may consider this extreme characteristic of applicationism [I_5] one of the most generic constraints that, appearing at the levels of *Society* and *School*, is based on not considering *mathematics as a fundamental tool for the search of answers to problematic questions that may appear in different fields of reality*.

Another important constraint shows up in the usual organisation of mathematical study programmes taught at natural science degrees. In fact, neither the structure given to the contents nor the way of developing them in class make it possible to carry out mathematical modelling work based on the study of *problematic questions* that arise in scientific fields which are close to the speciality chosen by the students. The 'raisons d'être', that is, the mathematical or extra-mathematical questions to which mathematical contents give answer, are not included in the study programme. The modelling activity is limited to the illustration of how certain pre-established models can be used to study certain standard pre-established problems.

In this line, we find some prefaces and content organisations in most of the books recommended for these courses, which helped us detect the presence of applicationism. As it is explained, the main aim of these books is to introduce students to a 'basic common language' for all scientists and in a completely independent way of natural sciences disciplines. For example, Salas & Hille (1995) explain it in the following terms:

> In this edition, you will find some easier applications to physics and, as extra chapters, some more difficult applications. […] Despite the incorporation of more applications, this book is still a mathematics book, not a science

> book or an engineering book. It is about calculus and its main basic ideas are limits, derivatives and integrals. The rest is secondary; the rest could be left out. (p.7)

The references to applications usually appear at the end of each chapter as an example of how to use the mathematical tools introduced. Often, these applications are included in a separate section labelled 'additional exercises', which offers an expansion of the field of problems that can be solved with the mathematical techniques introduced in that chapter [I_4].

We can consider that the lack of problematic questions to motivate the introduction of mathematical contents, is a clear consequence of the 'Euclidean style', if we use the term of Lakatos (1976), giving priority to the theoretical logic in the construction of concepts as opposed to the *heuristic* one, and which certainly underlies applicationism. It prevents didactic processes from focussing on the study of questions and it constitutes a clear constraint to the teaching of mathematics as a modelling process.

This idea, introduced by I_4, supports the progressive disappearance of a general scientific problem. Instead, there is a proliferation of isolated problems (arising from different systems). The interviews suggest a similar trend:

> A set of isolated and separated questions appears, and, I must add, it is a shame. But this is what is done. It is not only that there is no relationship between the instrumental courses, but also that there is no relationship between the descriptive courses! They are all satellites on their own isolated orbits […]. We could look for very serious reasons, but one always ends up finding the usual suspects, inertia and lack of unity.
>
> Each one seems to reign over his/her little kingdom, and explains the topics separately. The student will never have a global view, but I think that we, the teachers, do not have it either. There is no tradition of working in community, of meeting and agreeing on the content of each subject […] I do not see how it can be possible to avoid this tradition in the future, to escape from this inertia around us.

Constraints coming from the *dominant pedagogy* at university

In the same way as we have talked about a *dominant epistemology* at university level, we can also consider a *dominant pedagogy* related to the way this institution understands and organises the teaching and learning processes of any discipline (and not only mathematics). We have thus moved to the level of Pedagogy in the scale of levels of didactic codetermination, which, as we will see, restricts mathematical modelling activities in a way that mutually reinforces the influence of applicationism. At this moment, we do not have enough evidence to present a systematic and exhaustive characterization of the dominant pedagogy in Spanish natural sciences faculties. As research hypotheses, we will anyway consider three characteristics we postulate more influential in the way of organising teaching and learning processes at university level: *monumentalism*, *individualism*, and *protectionism*.

Monumentalism

A first important feature of the current dominant pedagogy at university level (at least in our experience of Spanish universities) is that the main objectives of the teaching processes are established in advance and formulated in terms of predetermined contents, instead of in terms of problems or questions to study. Furthermore, the organisation and validity of these contents are rarely questioned: they are supposed to be 'transparent' and not problematic. Once the contents are presented, problems or questions arise as a means to illustrate a specific functionality of the contents, the value of which is previously assumed. When a problem is studied, nobody discusses the possible existence of different answers from those previously established and provided by the teacher. It is not supposed necessary for the study community to check the validity and relevance of the answers proposed by the teacher. The only thing that can possibly be questioned is the particular way of organising teaching and learning processes, not the nature or validity of what is taught. This corresponds to the paradigm that Y. Chevallard (2006 2012) has named 'monumentalism', where contents seem to be considered as interesting monuments to visit, instead of useful tools to provide answers to problematic questions. A similar diagnosis can be found in the words of the former Under-Secretary-General of the United Nations, Hans van Ginkel (Ginkel 2001), who depicted this situation in the following terms:

> Another effort that has to be made regarding curricula concerns the problematisation of issues. All too often university courses limit themselves to themes […] without making the effort to inquire what issues form the core of these themes. I believe this lack of problematisation largely accounts for the slow progress of true understanding. Naming the theme and going for the easy answer tend to become in this context an institutional alibi – an alibi which does not necessarily serve the advancement of knowledge and research. The problematisation of issues also helps to bring the links between the different issues into focus. Without a clear understanding of these interlinkages, no effective understanding and management of issues will be possible.

In the case of mathematics for natural sciences this trait of 'monumentalism' of the dominant pedagogy is reinforced by what Lakatos (1976) present as the Euclidean or 'deductivist style' that, as we said before, seems to underlie applicationism:

> In deductivist style, all propositions are true and all inferences valid. Mathematics is presented as an ever-increasing set of eternal, immutable truths. Counterexamples, refutations, criticism cannot possibly enter. An authoritarian air is secured for the subject by beginning with disguised monster-barring and proof-generated definitions and with the fully-fledged

> theorem, and by suppressing the primitive conjecture, the refutation and the criticism of the proof. Deductivist style hides the struggle, hides the adventure. (p. 142)

This traditional way of presenting mathematical contents appears as an important constraint to the life of mathematical modelling since it practically eliminates the role and existence of successive provisional answers that constitute the core of any modelling process. In addition, school tradition supports the work with collections of texts where all these pre-established answers are inscribed. This tradition tends to cause a documentary shortage that ends up favouring work with very poor means for the pre-established study programmes.

Individualism

Another trait to mention is that, in our experience of Spanish universities, the dominant pedagogy supports individualized and strongly directed teaching. In other words, what prevails is the students' individual work under the orders of the teacher, thus hindering collective activities that could support the autonomous behaviour of a group of students in a classroom. On eliminating collective decisions and public discussions, the scientific school activity does not promote the role of the study community in the building of knowledge.

Individualism is a trait of the dominant pedagogy that seems to be coherent with the previous ones and, in particular, with the aforementioned monumentalism of the deductivist style. If the problems and questions which are at the origin of the construction of mathematical knowledge are forgotten and there is only a set of pre-established answers students must learn, then an individualized way of teaching seems to be the most appropriate and economic option. At the same time, individualism hinders collective activities such as discussing modelling procedures and defending different possible answers, which are not considered by Euclidianism as part of the scientific knowledge.

Protectionism

Finally, another hypothetical feature to characterise the dominant pedagogy in natural sciences university institutions, closely linked to the previous one is that, to prevent students from dropping out of school, the dominant pedagogy tends to eliminate all those aspects of mathematical discipline that are especially difficult and highly demanding. This 'protectionist' principle tends to progressively reduce long-term aims and, at the same time, the myth of an immediate and almost instantaneous comprehension gets stronger, which leads to weaken systematic and patient work. It also tends to atomize taught mathematics (and, in general, all teaching contents) into independent and small themes that can be learnt quite fast and more easily. All these traits of the dominant pedagogy represent important constraints on the life of mathematical modelling, which often requires a systematic, long-term and articulated activity.

To sum up, our perception of the dominant pedagogy in natural sciences university institutions, mutually reinforced by applicationism, is that it tends to separate mathematics from the rest of natural sciences disciplines and organises its teaching according to the internal logic of concepts (instead of those of problems). This dominant pedagogy, as we perceive it, eliminates the rationale of the taught mathematical praxeologies, presenting them as static monuments that must be studied without showing their functionality as tools to provide answers to problematic issues. In addition, all these pre-established contents are supposed to be transparent and not problematic, thus strongly restricting critical thought and public discussion. This is part of the visible resistance the university teaching system offers to the implementation of new proposals to introduce mathematics as a modelling activity. As we will see in the next section, when introducing a change in the system through the implementation of a new teaching activity, other oppositions come to light, many of which are related to the specific levels of didactic codetermination.

CREATING CONDITIONS FOR MATHEMATICAL MODELLING: STUDY AND RESEARCH PATHS

The study of the ecology of mathematical modelling in current university settings cannot be limited to the naturalistic observation of the teaching of mathematics in Spanish natural sciences university degrees (syllabi, textbook prefaces, interviews and surveys). It is also important to carry out research aiming at introducing new conditions for the 'normal life' of mathematical modelling activities. At the same time, this intervention in the current university educational system makes it react and can thus highlight new constraints, less visible through direct observation, some of them more closely related to the mathematical issues to be taught and that appear at the specific levels of didactic codetermination.

In the research here presented, the intervention consists in the local implementation of three connected *study and research paths* (SRP) about the dynamics of biological populations. As said before, the hypothesis underlying this implementation is the capacity for SRP to facilitate the inclusion of mathematical modelling as a regular activity in university settings (Barquero, Bosch & Gascón 2008).

A SRP can be described as a sequence of questions and answers, starting from a *generating question Q* and including both the construction of appropriate experimental *milieux* (in the sense of non-intentional systems, see Brousseau 1997), and the access to previously available knowledge through some *media* (Chevallard 2006). The notion of SRP is very general and includes any kind of inquiry process (scientific, journalistic, judicial, etc.) as well as the processes of direct teaching aimed at making information or knowledge available to a given group of persons (the students) (Chevallard 2012, see also Winsløw, Matheron & Mercier 2013).

A SRP on population dynamics

We tested the implementation of a SRP with first-year students of a technical engineering degree (a 3-year programme) in the Industrial Chemical Engineering department at the Universitat Autònoma de Barcelona during five academic years (from 2005/06 to 2009/10). The SRP took place within the one-year 'Mathematical Foundations of Engineering' course. A special educational activity, called the 'mathematical modelling workshop' was introduced in the general organisation of the course. It ran in parallel with the usual lecture sessions (three hours/week) and seminar or problem sessions (one hour/week) dedicated to present the main contents of the course and exemplify said contents by carrying out some exercises on the blackboard. The workshop consisted in 2-hour weekly sessions during most of the course. Attendance was optional for the students and it provided one extra point (out of 10) in the final assessment of the course. Two instructors took part in the course: the lecturer who was a researcher in applied mathematics and the instructor of the seminars, the first author of this paper, who was the person in charge of the workshop.

The workshop started from an initial question (that we will call Q_0) about how to *predict a population dynamics*. More concretely, given the size of a population over some previous periods of time, *how can we predict the long-term behaviour of its size? What sort of assumptions on the population, its growth and its surroundings should be made? How can one create forecasts and test them?* In all its implementation, the workshop focused on this initial problematic question Q_0 to which students had to build a complete answer along the entire academic year. Q_0 was presented using different populations: a pheasant population (see figure 2), then a fish population and, finally, a yeast population that was cultivated either in independent containers or together.

To provide some answers to Q_0 and to the sequence of *derived questions* that followed it, the construction of different mathematical models was required. Depending on whether time was considered as a discrete or continuous magnitude, *discrete* or *continuous* models were developed. If time t is measured in discrete units, and x_t depends, among other factors, on past states $x_{t-1}, x_{t-2}, \ldots, x_{t-d}$ $(0 < d \leq t)$, studying question Q_0 leads to considering two main types of models: when x_t only depends on x_{t-1} (population with independent generations), the models are based on recurrent sequences of first order: $x_{t+1} = f(x_t)$ where f is a one-variable real valued function. Otherwise, when x_t depends on $d > 1$ past generations $x_{t-1}, x_{t-2}, \ldots, x_{t-d}$ (population with mixed generations), the models are based on recurrent sequences of order $d > 1$, which can be expressed as *vector recurrent sequences* with $X_{n+1} = f(X_n)$ where $X_0 = (x_0, x_1, \ldots$

x_{d-1}) is the vector of the d initial generation sizes, $X_1 = (x_{d,} x_{d+1}, \ldots x_{2d-1})$ represent the d following generations, and $X_k = (x_{k.d,}\ x_{k.d+1}, \ldots, x_{(k+1).d-1})$ is the k-th vector of d generations, $0 \leq k \leq n$. In the first case, the models use most of the contents that usually integrate elementary calculus; the second kind of models, when f is a linear function, covers the field of linear algebra. If time t is measured as a *continuous variable*, the study focuses on the continuous evolution of the population, which has an analogous structure to the situations described above: models made of ordinary differential equations of order 1 (separate generations) or higher (mixed generations).

Humans have introduced many populations in new habitats, accidentally or intentionally, to study their dynamics. Some of them have become very interesting for ecological research. For example, in 1937, eight pheasants (*Phasianus colchicus torquatus*) were introduced in a protected island in front of Washington. There were enough resources in the island and no predators. Also, the island was far enough from the coast for the pheasants to fly out of it.

In the following table and its corresponding graph representation, there are some collected data on the evolution of the size of that population between 1937 and 1942, taken each year and at the same stage of the cycle life.

Year	Size of population
1937	8
1938	26
1939	85
1940	274
1941	800
1942	1800

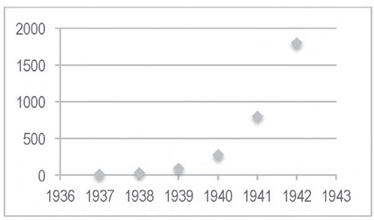

Figure 2. – Introductory worksheet to the mathematical modelling workshop, data provided by Lack (1967).

Due to official curricula conveniences, the initial question Q_0 can be divided into three sub-questions: discrete models with independent generation populations, discrete models with mixed generation populations and continuous models, which were respectively approached during each of the three terms of the course (see figure 3). At the end, this sequence of modelling activities covered most of the contents (one-variable calculus, linear algebra and ordinary differential equations and their systems) of a first-year course of mathematics for natural science students at university level.

In the consecutive academic years where the SRP was implemented, the workshop generally took place as follows: students worked in teams of 2 or 3 members, where they were asked to produce their own answer to Q_0 at the end of each term and at the end of the course. Among other tasks and responsibilities assigned, they had to consider intermediate (sub)questions and write, submit and defend a team report, in each weekly session, with their 'temporary' answers to these intermediate questions. At the beginning of each session, one team was in charge of explaining and defending their report. A discussion followed to state the main progress and to agree on how to continue the study process. At the end of the SRP, each student individually had to write a final report of the entire study (evolution of problematic questions, work in and with different models, relationship between them, etc.).

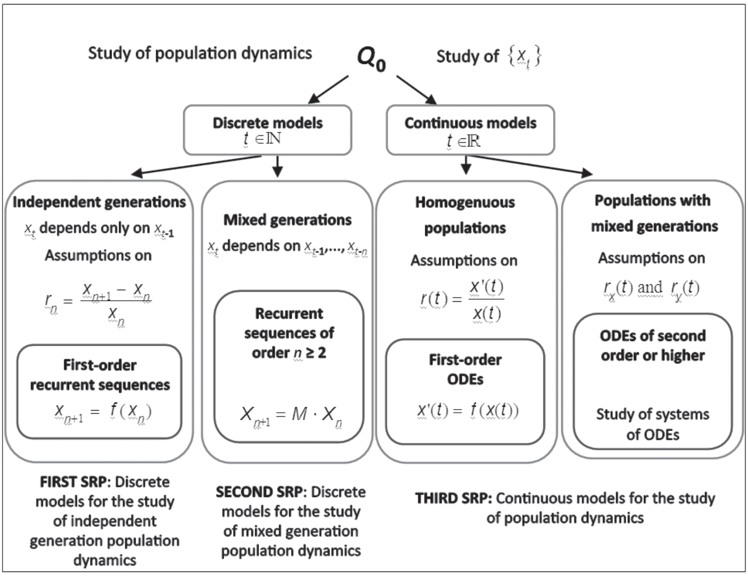

Figure 3. – General structure of the branches derived from the study of Q_0

After the implementation of the SRP with first-year university students along five academic years, a set of regularities or invariants appeared that helped us postulate a set of *conditions,* which enable their development as well as the set of *constraints* that limit it and could, in the long run, put its viability at risk. In what follows, we will introduce some *driving forces* of the dynamics of SRP and the correspondent *didactic activities* that come up to create appropriate *conditions* for their implementation—and, thus, for a possible integration of mathematical modelling processes at university level.

A generating question at the starting point of a SRP

The starting point of a SRP is a 'lively' question of genuine interest to the community of study, what we call the *generating question* of the study process, here denoted by Q_0. It should not be a question imposed by the instructor to cover some learning context fixed a priori. Elaborating answers for Q_0 has to become the main purpose and an end in itself. The study of Q_0, together with the *derived questions* that can appear throughout the study, is the *origin*, *engine* and *'raison d'être'* of the whole study process. In this sense, Q_0 should be present during the entire study process and acts as its articulating axis.

The case of the SRP on population dynamics provides a good example of the power of a generating question. In each term, different aspects that revolve around Q_0 were analysed: forecasting the size of the population in the short, medium and long term, considering time as a discrete variable (first-order sequences models, 1st term); the forecast in discrete time distinguishing two or three groups in a generation (models based on matrix algebra, 2nd term); and, the same forecast considering time as a continuous variable (differential equations and systems of equations, 3rd term). The possibility to divide the SRP into these three branches (figure 3), one for each term of the academic year, was extremely helpful since it respected the usual division of the syllabi into three 'blocks of contents': linear algebra, calculus and differential equations. Some of these intermediate questions (t as a discrete or continuous variable, long and short term predictions) were spontaneously considered by the students. Other (mixed generations) were introduced by the teacher, who also led the order of approaching them, to harmonise it with the structure of the syllabus.

With the implementation of the SRP, we tested how the sequence of questions arising from Q_0 led the students and the teacher to consider most of the main contents of the entire mathematics course, plus some additions (recurrent sequences, speed of convergence, graphical simulation, etc.). However, during each branch of the SRP, these contents appeared in a very different structure from the traditional organisation of the course: instead of the classical logic of mathematical concepts, the workshop was more guided by the need to give an answer to the problematic questions that appeared and by the types of models that progressively appeared to build this answer.

One of the biggest constraints that need to be faced is how to make SRP compatible with the traditional description of a syllabus. For instance, an important constraint was the difficulty of breaking the classical course structure of 'lectures/seminar/exams' to introduce the modelling workshop. In the successive implementations of the SRP, over the years, the workshop was more integrated with the rest of the educational activities. Lectures and seminars were used to provide students with some of the necessary tools to follow the workshop while, throughout the workshop, it was sometimes necessary to stop for some sessions or weeks to include some lectures that provided students with the appropriate tools to continue with the study of Q_0. For instance, in the second branch of the SRP with models based on matrix algebra, several sessions were used to introducing students to some techniques about matrices diagonalisation. And, vice versa, the workshop motivated and showed the functionality of the main content of the course: linear algebra, ODE, and so on. The integration of SRP in the regular organisation of the course and its linkage with the other activities (lectures, seminars, assessment) thus appeared as a necessary condition for its survival in the long run.

Tree structure of SRP

During a SRP, the study of the generating question Q_0 evolves and opens many other *derived questions* Q_1, Q_2, …, Q_n. One needs to constantly ask whether these derived questions are relevant. The fundamental criterion to decide whether they are indeed relevant is to ensure that they are capable of providing *temporary answers* A_i that can be helpful in elaborating a *final answer* $A^\heartsuit$ for Q_0. As a result, the study of Q_0 and its derived questions Q_i leads to successive temporary answers A_i tracing out the *possible routes* to be followed in the effective experimentation of the SRP. Thus, the work of production or construction of $A^\heartsuit$ can be described as a tree of questions Q_i and temporary answers A_i related to each other during a modelling process. In the case of the first branch of the SRP about 'Discrete' models for independent generation population dynamics', its tree-structure can be summarized in the following diagram in terms of questions and their successive answers (see fig. 4).

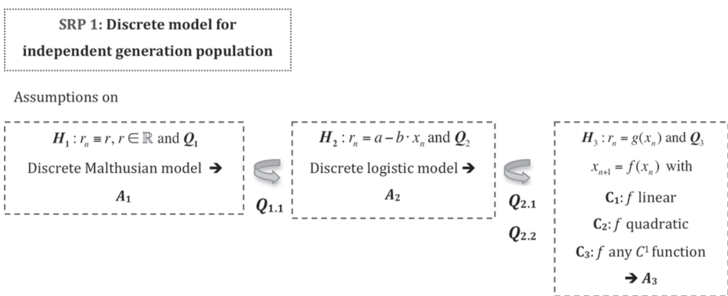

Figure 4. – Summary of the first branch of the SRP in terms of Q_i and A_i

In a SRP, when studying the links between Q_i and A_i provided by the models, new questions appeared that forced to broaden the previous models to more comprehensive, rich and complex ones, which made them continue with the modelling process that is both *progressive* (integration of models into more complex and general ones) and *recursive* (each model is included in the next one). For instance, in this first branch of the SRP, a first model appeared when students assumed that the *relative rate of growth* of the population could be considered as constant. It led to the construction of the discrete Malthusian model that provided students with the first temporary answers A_1 to Q_1. Students quickly realized with a new question $Q_{1.1}$ how to overcome the unrealistic fact assuming the existence of infinite resources. This new question, which showed the limitation of the first model, motivated the study community to expand the initial assumptions and consider a new broader and more complex model, which was the case of the logistic model. The numerical simulation with the logistic model opened new questions $Q_{2.i.}$ about the convergence of the population evolution that could not be answered with the tools previously used. It led to extend the model to a general functional model (which included the Malthusian and logistic models as particular cases) where new tools had to be considered: functions, graphical simulation, derivatives, etc. Thus, study and research paths enable to locate at the core of the study process the dynamics of the construction of knowledge through the use and development of different models, instead of directly presenting final answers to questions clearly formulated by the teacher. A new logic of the construction of mathematical knowledge appears besides the prevailing theoricist one and, in the long run, could end up challenging with it. Therefore, making this dynamics of the SRP visible seems to be an important condition to break down the monumentalism and applicationism of the dominant pedagogy and epistemology at this school level.

The dialectics of the *media* and the *milieux*

The implementation of a SRP can only be carried out if the students have some partial pre-established answers $A_i^{\lozenge}$ to the derived questions Q_i that appear throughout the study process, as well as some other objects O_j to test the available answers, elaborate new ones and formulate new questions. The pre-established answers $A_i^{\lozenge}$ are accessible through different means of communication and diffusion, which we call the *media*. *Media* are any source of information such as textbooks, treatises, research articles, class notes, etc. the intention of which is to provide knowledge to others. However, the knowledge provided by the *media* corresponds to constructions that have usually been elaborated to answer other questions than those concretely approached. Thus, it has to be 'deconstructed' and 'reconstructed' according to the new needs. In a SRP, it will then be necessary to dispose of an appropriate *experimental milieu*, *M*, which will evolve throughout the study process. This would start a dialectics—called the *media* and *milieu* dialectics—to test the temporary answers given by the media, against appropriate milieus, in order to obtain new knowledge to answer new questions.

This dialectics is an essential part of the mathematical modelling processes in the following sense: when constructing a model of a certain system, students need to access previously available knowledge that can be considered as partial answers to Q_0, not reduced to the 'official' and 'final' one. They also need empirical means (a *milieu*) to validate the partial answers or models proposed. In the SRP, students are systematically asked to look for information in the media about the possible types of models related to the systems considered. In particular, they have to look for already existing models and wonder whether they are important enough so as to be assigned a specific name, and so on. The validity of the models constructed is carried out from data—which in our case the teacher had provided—and through numerical simulation with a spreadsheet or a symbolic calculator.

Promoting the role of the community of study: the individual/social dialectics

A real integration of mathematical modelling as a regular activity into the classroom needs to *promote the role of the study community* guided by the teacher who assumes the role of *study director*. The study community has to be in charge of collectively studying Q_0 and producing an appropriate collective answer $A^♥$. In contrast with the dominant pedagogy where there is a dominance of individual students' work under the guidelines of the teacher, in a SRP the group of students and their study director have to share the set of tasks and negotiate the responsibilities to assume when carrying them out.

This shift from the *individual* to the *community* had many important consequences during the implementation of the SRP and to make the existence of mathematical modelling possible. In the SRP, the collective study of questions provides the community with the opportunity to *defend* their own answers, instead of accepting the *imposition* of the official ones. Unlike a teaching process addressed to individual learning where the interrelations and agreements among students are minimized, in a SRP students need to assume a lot of new responsibilities that the traditional *didactic contract* assigns exclusively to the teacher, for instance addressing questions, creating hypotheses, searching and discussing different ways of looking for an answer, comparing experimental data and reality, choosing the relevant mathematical tools, criticizing the scope of the models constructed, writing and defending reports with partial or final answers, etc. Given the fact that, as was said before, these actions are at the core of mathematical modelling, it seems clear that an educational strategy centred on individual learning in detriment of the collective construction and study of questions does not facilitate the development of modelling activities at school.

In the SRP whose implementation we report here, the teacher thus had to assume a new role of acting like the leader of the research and study process, instead of lecturing the students. It soon appeared that the teaching culture at university level does not offer a variety of teaching strategies for this purpose.

The dialectics of questions and answers as an engine of the SRP

As we mentioned earlier, an important dialectic integrated in the SRP is the task of posing questions and that of the search for answers. In the 'traditional' didactic contract, the charge of posing questions generally belongs to the teacher, while students only come up with doubts or questions that the teacher can give a quick answer to.

As we saw in the experimentation of the SRP, the mathematical modelling process required the entire community to focus on the study of a single question for a long period of time (the whole year!). This question had to remain 'alive' and 'open' session after session. Furthermore, the relevance of the derived questions and the opportunity of its consideration had to appear as one more gesture in the study process. It had to be negotiated between the teacher and the students.

This situation is rarely seen under the dominant pedagogy. For instance, this pedagogy only attributes to the teacher the ability to teach certain contents, the value of which nobody argues about. In order to overcome the constraints that appeared during the experimentation of the SRP (students' passiveness, their request for a close supervision by the teacher, etc.), the teacher introduced some new didactic devices: for instance, the teacher asked the students to formulate at least one new question that arose from the work carried out when handing in their weekly team report. Moreover, at the beginning of the following session these new questions were brought together and students–under the teacher's watchful eye–agreed on the way to continue. It was an excellent means to compare and discuss the work done during the entire process, and particularly, a way for the study community to formalise all the questions approached and their successive temporary answers.

Let us stress the importance and utility of the SRP *mathematical a priori design* to guarantee, on the one hand, that the generative question chosen is 'productive' enough to lead to many other derived questions. On the other hand, the design also gives a detailed description of the possible evolution of the study of Q_0 as a tree structure of potentially derived questions and their successive answers (Q_i, A_i) tracing out the possible routes to be followed (or, on the contrary, to be avoided) in the effective experimentation of the SRP.

The dialectics of the dissemination and reception of answers

In the current didactic contract, the teacher is usually tempted to close the process of study of problematic questions by proposing some pre-established answers, those that are broadly accepted within the educational institution (monumentalism's constraints). In a SRP, in contrast, the students have to defend the final answer $A^{\heartsuit}$ and also the successive answers A_i they provide, although they may still be of a temporary nature. Specific conditions are thus necessary since we cannot expect this to happen spontaneously.

In the case of our experimentation, as we mentioned before, these conditions consisted in the introduction of specific didactic devices or artefacts. The first one is what we named 'Report on results', relatively alien to the mathematical teaching culture at university level. Each week, in groups, the students had to elaborate a written text in which they gathered both the documents provided by the teacher and the partial results of the work done in the workshop session. They complemented it with their personal comments and the information on the subject they gathered. They had to submit the report to the teacher. These reports contained the answers that each group defended in class at the beginning of each session. The second device consisted in the writing of a 'Final report' each student had to submit at the end of the workshop. This report no longer contained the chronicle of the study process but focussed on presenting and defending a final answer to the question initially posed.

Undoubtedly, the students did not easily accept elaborating, reading and defending the reports due to the difference of contract as compared to other study devices used in other subjects. Despite all the initial resistance to the changes introduced during the implementation of SRP (working in groups, scheduling the study on their own, formulating questions, selecting mathematical contents, using a computer and bibliographical resources, writing and defending temporary answers, etc.), all these responsibilities traditionally assumed by teachers were progressively accepted by the students. This increasing autonomy taken on by the students during the SRP seems to be an important condition to carry out the activity of mathematical modelling.

CONCLUSIONS: THE SRP ECOLOGY

Our starting point was the problem of the large-scale dissemination of mathematical modelling in current university teaching. We carried out an analysis of the ecological dimension of mathematical modelling activities, using both the *naturalistic* and the *interventionist* methodologies: programmes, textbooks, interviews to natural science teachers on the one hand, and successive implementations of SRP on the other. This ecological analysis helps identify different constraints at different levels of generality, some of them closely related to the *dominant epistemology* (‘applicationism’) and to the dominant *pedagogy* (‘monumentalism’, ‘individualism’, ‘protectionism’). The experimentation of *study and research paths* during five academic years in a university setting has then shown the possibility to locally overcome some of these constraints and locate mathematical modelling at the core of the teaching process. This was carried out throughout the implementation of specific conditions that can also be considered as driving forces of SRP.

(1) The first one is the fecundity of the initial generating question. Placing the study of a ‘lively’ *generating question* Q_0, in our case on population dynamics, at the starting point of the SRP shows how the sequence of derived questions arising from Q_0 can led the students and the teacher to consider many contents of the entire mathematics course. The use, *raison d’être* and function of all these mathematical concepts and models appears to allow both, students and teachers, to provide answers to the problematic questions that appeared along the study. (2) The *tree structure* of the SRP and the dynamics and *dialectics between questions and answers* open a new logic of the construction and use of mathematical knowledge. It contrasts with the prevailing monumentalism and applicationism of the dominant pedagogy and epistemology at university level, which tends to presents mathematical knowledge as a sequence of static and definitive entities. (3) Closely

related to the previous conditions emerges the *media-milieu dialectic* based on the interaction of an appropriate *experimental milieu*, which evolve throughout the study process, and the temporary answers given by the *media*. (4) SRPs also support the *role of the study community* guided by the teacher who has to assume the new role of study director. Besides the dominance at university level of the individual student's work under the supervision of the teacher, SRP proposes that the group of students and their study director share the continuous looking for answers to the successive questions and negotiate the responsibilities to assume when carrying them out. (5) Finally, this shift from the *individual* to the *social* dimension has many important consequences in new roles assigned to students and teachers. A new didactic contract is required that, in contrast to the prevailing traditional didactic contract at university level, includes many responsibilities to students, traditionally assumed by the teacher: the responsibility of posing questions, of elaborating, presenting and defending their own answers, scheduling the study, among many others.

Given the nature of the constraints identified by the ecological analysis, if the main aim is to create the *appropriate conditions* for mathematical modelling to exist as a regular activity, it seems very difficult to directly modify the *dominant epistemology and pedagogy* in university teaching institutions. This is an almost impossible task for the institutional position of the 'teacher' (considered individually), and also for the community of researchers. Actually, it would imply to directly control some conceptions that are anchored in deep-rooted practices and in a strongly established culture in society.

The implementation of SRP thus proposes a bottom-up strategy, which, in a sense, is the opposite: beginning to set up appropriate local conditions to change school activities, initially respecting the traditional structure of lectures-seminar-exam with the incorporation of a mathematical modelling workshop making the carrying out of SRP possible. We observed how this strategy changed the global organisation of the course (lectures and seminars becoming more and more dependent of the workshop) and we postulate that some changes in the study processes, because they cause significant modifications in school mathematics, could favour the emergence of new mid-term epistemological and pedagogical models strengthening the development of a more 'functional' (as opposed to 'formal') mathematical activity.

Other investigations (García et al. 2006, Hansen & Winsløw 2010, Llanos & Otero 2013, Matheron 2010, Rodríguez, Bosch & Gascón 2008; Ruiz-Munzón, Matheron, Bosch & Gascón 2012, Sierra, Bosch & Gascón 2011) have shown a similar integrative potential of SRP in different school levels, mathematical contents and even in teachers' professional development. In all these investigations, SRP appear as a possible reaction to monumentalism and a way to introduce a functional relationship to mathematics. In addition, and even if not all these works approach it directly, the problem of the conditions fostering and the constraints hindering the regular existence of this new teaching and learning activity cannot be neglected.

The research study presented in this paper is the first attempt of a systematic analysis of the ecology of mathematical modelling and *study and research paths* using the scale of levels of didactic codetermination. We have seen in what sense Study and Research Paths can be a local possible answer to the problem of the difficult and tough ecology of mathematical modelling in current university teaching systems. We have shown some specific levels where action is possible, in the sense that new teaching conditions can be created through a specific work of *didactic engineering* (Margolinas et al. 2011); we have also pointed out some more generic levels that seem difficult to directly modify. In any case, the relationships between the different generic and specific constraints and, more particularly, the description of their mutual dependence, are very little known and will require more effort in future research. It seems especially important to take into account the potentials offered by teachers' professional development in the evolution of this ecology and, more particularly, in the evolution of epistemological and pedagogical dominant models.

ACKNOWLEDGEMENTS

Funded by the Spanish R&D projects EDU2012-39312-C03-01, EDU2012-39312-C03-03 and EDU2012-32644.

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